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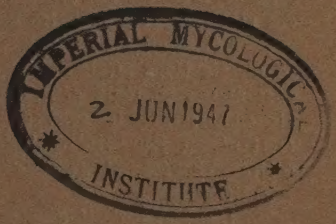

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MATHEMATICS

DEPENDENCE OF SOLUTIONS OF CAUCHY'S PROBLEM ON INITIAL CONDITIONS

By I. PETROVSKY

(Communicated by A. N. Kolmogoroff, Member of the Academy, 17.XII.1942)

Let a linear hyperbolic equation

$$\frac{\partial^n u}{\partial t^n} = \sum_{\substack{k_0<n \\ k_0+k_1+\ldots+k_p\leqslant n}} a_{k_0k_1+\ldots+k_p}(t, x_1, \ldots, x_p)\,\frac{\partial^{k_0+k_1+\ldots+k_p} u}{\partial t^{k_0}\,\partial x_1^{k_1}\ldots\partial x_p^{k_p}} \qquad (1)$$

be given. The coefficients $a_{k_0k_1\ldots k_p}(t, x_1, \ldots, x_p)$ are supposed to be sufficiently smooth. The lateral surface of the characteristic cone with the apex at the point $(t^*, x_1^*, \ldots, x_p^*)$ decomposes its base on the plane $t=t_0$ of initial data into some partial domains. One of such domains G_{t_0} will be called lacuna, if the solution of Cauchy's problem rests invariant at the point $(t^*, x_1^*, \ldots, x_p^*)$ under any variation of initial data within G_{t_0} only, provided that they remain sufficiently smooth. To make the picture definitive assume $t_0 < t^*$.

If t_0 is sufficiently near t^*, then to every domain $G_{t_0}^*$ produced by the lateral surface of the characteristic cone with the apex at $(t^*, x_1^*, \ldots, x_p^*)$ for the equation with constant coefficients

$$\frac{\partial^n u}{\partial t^n} = \sum_{\substack{k_0<n \\ k_0+k_1+\ldots+k_p=n}} a_{k_0k_1\ldots k_p}(t^*, x_1^*, \ldots, x_p^*)\,\frac{\partial^n u}{\partial t^{k_0}\,\partial x_1^{k_1}\ldots\partial x_p^{k_p}} \qquad (2)$$

on the plane $t=t_0$ there corresponds a unique domain G_{t_0} that is near $G_{t_0}^*$ and is constructed for the equation (1).

Theorem 1. *If for any t_0 sufficiently near t^* the domain G_{t_0} corresponding to $G_{t_0}^*$ is a lacuna for the equation* (1), *then $G_{t_0}^*$ is a lacuna for the equation* (2).

The purpose of the subsequent is to study the lacunae for linear hyperbolic equations of the form (2) with constant coefficients.

If the initial data

$$\frac{\partial^k u}{\partial t^k} = \varphi_k(x_1, \ldots, x_p), \quad k=0, 1, \ldots, n-1,$$

are different from 0 only within one of the domains $G^*_{t_0}$, the solution of Cauchy's problem may be represented in the form

$$u(t, x_1, \ldots, x_p) = \int \sum_s K_s(t-t_0, x_1-\xi_1, \ldots, x_p-\xi_p)\, \varphi_s(\xi_1, \ldots, \xi_p)\, d\xi_1 \ldots d\xi_p.$$

It is obvious that the investigation of lacunae is thus reduced to the determination of domains, in which the K_s equal 0. We shall study the problem of existence of stable lacuna, i. e. such that «endure» all suffuciently small variations of the coefficients $a_{k_0 \ldots k_p}$. We first assume that $n < p+1$. Generalizing and modifying slightly Herglotz's formulae (1), we can obtain the following results. Consider the algebraic surface of n-th degree

$$1 = \sum_{k_0+k_1+\ldots+k_p=n} a_{k_0 k_1 \ldots k_p} z_1^{k_1} \ldots z_p^{k_p} \tag{3}$$

where

$$a_{k_0 \ldots k_p} = a_{k_0 \ldots k_p}(t^*, x_1^*, \ldots, x_p^*).$$

In what follows we shall suppose that this surface possesses no singular points.

To determine $K_s(t, y_1, \ldots, y_p)$ consider the section of this surface by the plane

$$t + y_1 z_1 + \ldots + y_p z_p = 0. \tag{4}$$

We take an arbitraty real point A on the plane (4) and, drawing through it all possible straight lines with real coefficients, consider their points of intersection with the surface (3), lying on the plane (4).

The real points of intersection, as well as the imaginary ones, form some $(p-1)$-dimensional cycles that will be denoted by C_{real} and C_{imag}, respectively. It is evident that C_{real} does not depend on the choice of A; as to C_{imag} it is replaced by a homologous cycle at the variation of A. For any s $K_s(t, y_1, \ldots, y_p)$ can be represented either as Abelian integrals extended over C_{real} or C_{imag} (or over some homologous cycles), according as p is odd or even, or as derivatives of such integrals with respect to t.

Hence follows

Theorem 2. *In order that there exist a stable lacuna for the equation* (2) *in case* $n < p+1$, *it is necessary and sufficient that the cycles* C_{real} (*when* p *is odd*) *or* C_{imag} (*when* p *is even*), *corresponding to its points, be homologous to zero in the complex intersection of* (3) *and* (4).

There are examples of such lacunae.

To determine $K_s(t, y_1, \ldots, y_p)$ in case $n \geqslant p+1$ it is necessary to construct the cycles $C_{\text{real}}(\tau)$, when p is odd [resp. $C_{\text{imag}}(\tau)$, when p is even] for every plane

$$\tau + y_1 z_1 + \ldots + y_p z_p = 0, \quad 0 \leqslant \tau \leqslant t. \tag{5}$$

Then $K_s(t, y_1, \ldots, y_p)$ can be represented as Abelian integrals extended over the surface S formed by these cycles.

Theorem 3. *In order that ther exist a stable lacuna for the equation* (2) *in case* $n \geqslant p+1$, *it is necessary and sufficient that the cycles* $C_{\text{real}}(t)$ *or* $C_{\text{imag}}(t)$ *corresponding to its points, when* p *is odd or even, respectively, be homologous to zero.*

Suppose the condition to be satisfied. We stretch films on the cycles $C_{\text{real}}(\tau)$ [resp. $C_{\text{imag}}(\tau)$] in the complex intersections of (3) with the planes (5) for $\tau = t$ and $\tau = 0$. That is possible, because, as is easily seen, the cycle $C_{\text{real}}(0)$ [resp. $C_{\text{imag}}(0)$] is homologous to zero. Then the surface S can

be complemented to a cycle $\sum$, and $K_s(t, y_1, \ldots, y_p)$ can be represented as an Abelian integral extended over this cycle.

Hence follows

Theorem 4. *In order that there exist a stable lacuna for the equation* (2) *in case* $n \geqslant p+1$, *it is necessary and sufficient that the cycles* $\sum$ *corresponding to its points be homologous to zero in the complex surface* (3).

We succeeded in constructing examples, for which this condition is fulfilled, only when p is even.

The results of this note can easily be generalized for hyperbolic systems of equations.

Mathematical Institute.
Moscow State University.

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MATHEMATICS

ON THE THEORY OF BICOMPACT EXTENSIONS OF TOPOLOGICAL SPACES

By N. A. SHANIN

(Communicated by A. N. Kolmogoroff, Member of the Academy, 1.VI.1942)

§ 1. General Remarks. We use here the terminology and notations introduced in our papers (1) and (2).

Applying the notion of extension introduced by E. Čech (3) to the completely regular T-spaces (2), we shall say that S is a β-extension (or Čech's extension) of the completely regular T-space R, if the following conditions are fulfilled: 1) S is a bicompact completely regular T-space; 2) R is a subspace of S; 3) $\overline{R}^S = S$; 4) if $x \in S - R$, then $(x) \in \mathfrak{F}_S$; 5) every continuous and bounded in R function can be continuously extended on S. For any completely regular T-space R there exists the β-extension of R that is unique up to a homeomorphism, leaving invariant all points of R. It is not difficult to show that the condition 5) may be replaced by the condition 5'). If $\{F, G\}$ is a canonical pair of sets with respect to the family $\mathfrak{F}_R$ (2), then $\overline{F}^S \subset \eta_S(G)$. In the subsequent we shall use the following theorem due to A. Alexandroff (4):

A. Alexandroff's Theorem*. *The T-space S is the β-extension of a completely regular T-space R if and only if S is the $(\omega, \mathfrak{F}_R^+)$-extension of R.*

Descriptive properties of the $(\omega, \mathfrak{F}_R^+)$-extension of R (1) yield a new inner characteristic of β-extensions of completely regular T-spaces.

§ 2. Weight of $(\omega, \mathfrak{B})$-Extensions of the T-space R. Suppose that E is a T-space, $\mathfrak{P} \subset \mathfrak{F}_E$, $\mathfrak{Q} \subset \mathfrak{F}_E$. We shall say that the family $\mathfrak{Q}$ is a closed basis of the family $\mathfrak{P}$, if every element of $\mathfrak{P}$ is representable as the intersection of some elements of $\mathfrak{Q}$. $\mathfrak{Q}$ will be called the accurate closed basis of the family $\mathfrak{P}$ if: 1) $\mathfrak{Q}$ is a closed basis of $\mathfrak{P}$; 2) for any pair of sets $P \in \mathfrak{P}$, $P' \in \mathfrak{P}$ with $PP' = \Lambda$ there exist $Q \in \mathfrak{Q}$ and $P'' \in \mathfrak{P}$ such that $P' \subset Q \subset P''$ and $PP'' = \Lambda$ [condition 2) is equivalent to 2'): for any $P \in \mathfrak{P}$ and $H \in E\{\mathfrak{P}\}$ with $P \subset H$ there exist such $H' \in E\{\mathfrak{P}\}$ and $G \in E\{\mathfrak{Q}\}$ that $P \subset H' \subset G \subset H$]. The family $\mathfrak{A}$ will be called the accurate closed basis of the T-space E, if $\mathfrak{A}$ is an accurate closed basis of the family $\mathfrak{F}_E$. It is obvious that the closed basis $\mathfrak{A}$ of the T-space E is an accurate closed basis of E if and only if every open neighbourhood G of an arbitrary closed set F contains a neighbourhood $H \in E\{\mathfrak{A}\}$ of F.

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* A. Alexandroff considers completely-regular T_1-spaces, but the T_1-axiom is not essential. Since $\mathfrak{F}_R^+$ is a weakly regular family, it follows from A. Alexandroff's theorem that the T_1-axiom is fulfilled in the β-extension of a completely regular T_1-space R. Here and in the following $\mathfrak{F}_R^+$ denotes the family of all completely closed sets in the T-space R.

By the accurate weight of the family $\mathfrak{A} \subset \mathfrak{F}_E$ we mean the minimal power of accurate closed bases of the family $\mathfrak{A}$. The accurate weight of the family $\mathfrak{F}_E$ will be called the accurate weight of the T-space E.

Theorem 1. *If the T-space R has the infinite weight, then the weight of the $(\omega, \mathfrak{B})$-extension of R is equal to the accurate weight of the family $\Delta\mathfrak{B}$. In particular, the weight of the ω-extension* of R is equal to the accurate weight of R, and the weight of the β-extension of the completely regular T-space R coincides with the accurate weight of the family $\mathfrak{F}_R^*$.*

The theorem follows from the following lemmas:

Lemma 1. *If $\mathfrak{P}$ and $\mathfrak{Q}$ are closed bases of a bicompact T-space E, then $\Delta\mathfrak{Q}$ is an accurate closed basis of the family $\Delta\mathfrak{P}$; in particular, if $\mathfrak{A}$ is a closed basis of the T-space E, then $\Delta\mathfrak{A}$ is an accurate closed basis of E.*

Lemma 2. *Let S be an $(\omega, \mathfrak{B})$-extension of a T-space R and $\mathfrak{E} \subset \mathfrak{F}_R$. In order that the family $\mathfrak{E}$ be a closed framework of S in R, it is necessary and sufficient that $\Delta\mathfrak{E}$ be an accurate closed basis of the family $\Delta\mathfrak{B}$.*

Corollary of Theorem 1 [B. Pospišil's Theorem (5)]. *The weight of the ω-extension (β-extension) of an infinite isolated** T-space of power $\mathfrak{n}$ is equal to $2^{\mathfrak{n}}$.*

§ 3. Conditions of Coincidence of $(\omega, \mathfrak{B})$ and $(\omega, \mathfrak{E})$-Extensions of R. Suppose that E is a T-space, $\mathfrak{P} \subset \mathfrak{F}_E$, $\mathfrak{Q} \subset \mathfrak{F}_E$. We shall say that the family $\mathfrak{P}$ is subject to the family $\mathfrak{Q}$ if, whatever be a finite system $\{P_k\}_{k=1}^n$ of elements of $\mathfrak{P}$ and the sets $F \in \mathfrak{Q}$, $G \in E\{\mathfrak{Q}\}$ such that $\prod_{k=1}^{n} P_k \subset F \subset G$, there exists a system $\{Q_k\}_{k=1}^n$ of elements of $\mathfrak{Q}$ with $P_k \subset Q_k$ $(k = 1, 2, \ldots, n)$ and $\prod_{k=1}^{n} Q_k \subset G$. If $\mathfrak{P} \subset \mathfrak{Q}$, then evidently $\mathfrak{P}$ is subject to $\mathfrak{Q}$.

Theorem 2. *Suppose that $\mathfrak{B}$ and $\mathfrak{E}$ are reduced closed bases of the T-space R, and S is the $(\omega, \mathfrak{B})$-extension of R. In order that S be at the same time the $(\omega, \mathfrak{E})$-extension of R, it is necessary and sufficient that: 1) $\Delta\mathfrak{E}$ be an accurate closed basis of the family $\Delta\mathfrak{B}$ and 2) the family $\Delta\mathfrak{E}$ (or, which is equivalent, the family $\mathfrak{E}$) be subject to the family $\Delta\mathfrak{B}$.*

This theorem follows from Lemma 2 and

Lemma 3. *Suppose that S is an $(\omega, \mathfrak{B})$-extension of the T-space R and $\mathfrak{E} \subset \mathfrak{F}_R$. The family $\mathfrak{E}$ is Π-separated in S if and only if $\Delta\mathfrak{E}$ (or, which is equivalent, $\mathfrak{E}$) is subject to the family $\Delta\mathfrak{B}$.*

Corollary of Theorem 2. *An $(\omega, \mathfrak{B})$-extension of the T-space R (in particular, the β-extension of the completely regular space R) is the ω-extension of R if and only if $\Delta\mathfrak{B}$ (resp. $\mathfrak{F}_R^*$) is an accurate closed basis of R.*

Theorem 3. *In order that the β-extension S of the completely regular T-space R be at the same time the ω-extension of R, it is necessary and sufficient that R be a normal T-space.*

The sufficiency of the condition in the case when R is a T_1-space was established by P. Alexandroff (6). Theorem 3 follows from Corollary of Theorem 2 as well as from

Lemma 4. *The completely regular T-space R is normal if and only if the family $\mathfrak{F}_R^*$ is an accurate closed basis of R.*

§ 4. Inner Characteristics of Topologically Complete Spaces. Theorem 4. *Suppose that R is a T-space, $\mathfrak{B}$ is a reduced closed basis of R and $\mathfrak{n}$ is a cardinal number (or an integer). The set R is representable as the intersection of at most $\mathfrak{n}$ sets that are open in the $(\omega, \mathfrak{B})$-extension of R if and only if there exists a family $\mathfrak{N}$ of vanishing ω-families in $\mathfrak{B}$ of power $\leqslant \mathfrak{n}$ possessing the property: for any maximal vanishing*

---

* According to the definition given in (1), the T-space S is the ω-extension of R if it is the $(\omega, \mathfrak{F}_R)$-extension (or H. Wallman's extension) of R.

** A T-space is called isolated if all its subsets are closed (and open).

ω-family Z in $\mathfrak{B}$ $\mathfrak{R}$ contains such an X that $X \subset Z$ (or, in other words, for any vanishing ω-family Y in $\mathfrak{B}$ there exists $X \in \mathfrak{R}$ such that $X+Y$ is an ω-family in $\mathfrak{B}$).

We shall outline the proof of the theorem.

Necessity. Let S be an $(\omega, \mathfrak{B})$-extension of R and $R = \prod_{Q \in \mathfrak{Q}} Q$, where $\mathfrak{Q} \subset G_S$ and the power of $\mathfrak{Q}$ does not exceed $\mathfrak{n}$. To every $Q \in \mathfrak{Q}$ different from S we put in correspondence the family

$$X_Q = \underset{F}{\mathcal{E}} (F \in \mathfrak{B} \quad \text{and} \quad S - Q \subset \overline{F}^S)$$

and set: $\mathfrak{R} = \{X_Q\}_{Q \in \mathfrak{Q}}$. $\mathfrak{R}$ is then the required family of vanishing ω-families in $\mathfrak{B}$.

Sufficiency. Let S be an $(\omega, \mathfrak{B})$-extension of R. For every $X \in \mathfrak{R}$ we put $Q_X = S - \prod_{F \in X} \overline{F}^S$. If $\mathfrak{Q} = \{Q_X\}_{X \in \mathfrak{R}}$, then $\mathfrak{Q} \subset \mathfrak{G}_S$, the power of $\mathfrak{Q}$ does not exceed $\mathfrak{n}$ and $R = \prod_{Q \in \mathfrak{Q}} Q$.

Let $\mathfrak{K}$ be an open basis of the T-space R. The family $\mathfrak{U}$ will be called the irreducible covering of R in $\mathfrak{K}$ if: 1) $\mathfrak{U} \subset \mathfrak{K}$; 2) $\mathfrak{U}$ is a covering of the set R; 3) $\mathfrak{U}$ contains no finite coverings of R. We shall say that $\mathfrak{U}$ is a maximal irreducible covering of R in $\mathfrak{K}$ if $\mathfrak{U}$ is an irreducible covering of R in $\mathfrak{K}$ and is not contained in any other irreducible covering of R in $\mathfrak{K}$.

Theorem 5. *Suppose that R is a T-space, $\mathfrak{B}$ is a reduced closed basis of R and $\mathfrak{n}$ is a cardinal number (or an integer). The set R is representable as the intersection of at most $\mathfrak{n}$ sets that are open in the $(\omega, \mathfrak{B})$-extension of R if and only if there exists a family $\mathfrak{R}$ of irreducible coverings of R in $R\{\mathfrak{B}\}$ of power $\leqslant \mathfrak{n}$ possessing the property: for any maximal irreducible covering $\mathfrak{W}$ of R in $R\{\mathfrak{B}\}$ $\mathfrak{R}$ contains a $\mathfrak{U}$ such that $\mathfrak{U} \subset \mathfrak{W}$ (or, in other words, for any irreducible covering $\mathfrak{V}$ of R in $R\{\mathfrak{B}\}$ there exists such $\mathfrak{U} \in \mathfrak{R}$ that $\mathfrak{U}+\mathfrak{V}$ is likewise an irreducible covering of R in $R\{\mathfrak{B}\}$).*

Following E. Čech [3], we call the completely regular T_1-space R topologically complete if it is $\mathfrak{G}_\delta$ in one of its bicompact extensions. As it was established by Čech, the necessary and sufficient condition that R be topologically complete is that R be $\mathfrak{G}_\delta$ in its β-extension. Čech further proved that a metrizable T-space is completely metrizable if and only if it is topologically complete. Keeping this in mind, we see that Theorems 4 and 5, being applied to the case $\mathfrak{n} = \aleph_0$, $\mathfrak{B} = \mathfrak{F}_R^*$ (or to the case $\mathfrak{n} = \aleph_0$, $\mathfrak{B} = \mathfrak{F}_R$, when the T-spaces are metrizable and in virtue of the normality their ω- and β-extensions coincide), give the inner characteristics of topologically complete and completely metrizable T-spaces.

Institute of Mathematics and Mechanics.
Leningrad State University.

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MATHEMATICS

ON SOME PROBLEMS OF THE FUNCTIONAL ANALYSIS

By V. ŠMULIAN

(Communicated by S. L. Soboleff, Member of the Academy, 28.XI.1942)

Let E be an arbitrary Banach space (1). Denote by $\overline{E}$ the space adjoint to E. The set $\overline{E}$ considered with the «weak» topology with respect to E is a linear topological space ε_* (2).

The following hypotheses were recently suggested by the author.

A. *If a bounded set $\mathfrak{M} \subset \overline{E}$ is closed in ε_* and the linear functional $F(f) \in \overline{E}$ is continuous on $\mathfrak{M} \subset \varepsilon_*$, then for any $\varepsilon > 0$ there exists an element $x_\varepsilon \in E$ such that*

$$|F(f) - f(x_\varepsilon)| \leqslant \varepsilon \qquad (f \in \mathfrak{M}).$$

B. *Let Q be a bicompact Hausdorff space. Suppose that a partially ordered sequence of continuous functions $\{\varphi_\alpha(q)\}$, $|\varphi_\alpha| \leqslant 1$, converges to zero at every point $q \in Q$. Then*

$$f(\varphi_\alpha) \to 0$$

for any linear functional f in the space of continuous functions on Q.

Theorem A was proved by the author for a variety of particular cases. Theorem B is true, provided that the partially ordered sequence $\{\alpha\}$ is simply the sequence $\{1, 2, \ldots\}$. But, as will be proved in the present paper, Theorems A and B are not true in general.

§ 1. Let us choose n arbitrary points

$$t_1 < t_2 < \ldots < t_n$$

on the segment [0,1]. The system of these points will be denoted by the symbol

$$\alpha = \alpha(t_1, t_2, \ldots, t_n).$$

We write

$$\alpha' = \alpha'(t'_1, \ldots, t'_{n'}) \leqslant \alpha'' = \alpha''(t''_1, \ldots, t''_{n''})$$

if every t'_i is among t''_j, but not conversely.

The symbols $\{\alpha\}$ constitute then a partially ordered sequence. For any α a continuous function $\varphi_\alpha(t)$ can be chosen with the properties:

1) $0 \leqslant \varphi_\alpha(t) \leqslant 1 \quad (0 \leqslant t \leqslant 1)$

2) $\int_0^1 \varphi_\alpha(t)\, dt = \frac{1}{2}$

3) $\varphi_\alpha(t) = 0$, whenever $\alpha = \alpha(\ldots t \ldots)$.

Then we have $\|\varphi_\alpha\| \leqslant 1$, $\varphi_\alpha(t) \to 0 \quad (0 \leqslant t \leqslant 1)$, $\int_0^1 \varphi_\alpha(t)\,dt \to \frac{1}{2}$.

This simple example shows that Theorem B is not true in general. In this connexion the problem arises:

For what kind of systems $\{\alpha\}$ *Theorem B is true?*

It is possible to show that it is true, when the system $\{\alpha\}$ consists of an enumerable set of symbols*.

Suppose that we have an arbitrary partially ordered sequence of bounded functions $\psi_\alpha(t)$ defined on a abstract set T.

We shall say that $\psi_\alpha \to 0$ quasi-uniformly on T if for any $\varepsilon_0 > 0$ and α_0 there exist such $\alpha_1, \ldots, \alpha_p > \alpha_0$, that

$$\inf_{1 \leqslant i \leqslant p} |\psi_{\alpha_i}(t)| \leqslant \varepsilon_0 \quad (t \in T).$$

The subset $\{\alpha'\} \subset \{\alpha\}$ will be called the directed regular part of $\{\alpha\}$ if for any α there exists an $\alpha' > \alpha$.

Theorem 1**. *Let* $\psi_\alpha(t)$ *be an arbitrary partially ordered sequence of bounded functions defined on an abstract set* T. *In order that*

$$\psi_{\alpha'}(t) \to 0$$

quasi-uniformly for every directed regular part $\{\alpha'\} \subset \{\alpha\}$, *it is necessary and sufficient that*

$$\lim_{\alpha} \lim_{\beta} |\psi_\alpha(t_\beta)| = 0 \tag{1}$$

for any partially ordered sequence $\{t_\beta\} \subset T$.

A partially ordered sequence of bounded functions $\{\psi_\alpha(t)\}$ is said to converge weakly to zero if

$$f(\psi_\alpha) \to 0$$

for all linear functionals f in the space of bounded functions $\psi(t)$.

The following theorem is valid:

Theorem 2. *If* $\psi_\alpha \to 0$ *weakly, then* (1) *holds. Conversely, if Theorem* B *is valid for the system* $\{\alpha\}$, *then every uniformly bounded partially ordered sequence satisfying* (1) *converges weakly to zero****.

§ 2. Consider the system of functions

$$\varphi'_\alpha(t) = 1 - 2\varphi_\alpha(t), \qquad (0 \leqslant t \leqslant 1),$$

where φ_α are continuous functions satisfying the conditions 1), 2), 3) of § 1. Then

$$\int_0^1 \varphi'_\alpha(t)\,dt = 0,$$

$$\varphi'_\alpha(t) \to 1, \quad \|\varphi'_\alpha\| \leqslant 3, \quad (0 \leqslant t \leqslant 1),$$

$$\int_0^1 1\,dt = 1.$$

We see that not only Theorem B, but the following assertion is not true:

C. *Let* Q *be a bicompact Hausdorff space. If a partially ordered sequence of*

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* This can be proved by a slight modification of L. Kantorovitch's proof of the theorem on passing to the limit under the sign of the integral.

** An analogous fact for ordinary sequences was first proved by L. Kantorovitch and G. Fichtenholz.

*** See the analogous theorem due to S. Banach [1].

continuous functions $\{\varphi_\alpha(q)\}$, $\|\varphi_\alpha\| \leqslant 1$ *converges to a continuous function* $\varphi_0(q)$ $(q \in Q)$, *then the function* φ_0 *belongs to the closed linear hull over the set* $\{\varphi_\alpha\}$.

We now shall show that Theorem C is a corollary of Theorem A, and thus it will be proved that Theorem A is not true.

Denote by E the closed linear hull over the set of functions $\{\varphi_\alpha\}$. Every point $q \in Q$ generates a linear functional

$$f_q(\varphi) = \varphi(q) \qquad (\varphi \in E)$$

in the space E. The functionals $\{f_q; q \in Q\}$ constitute a bounded set $\mathfrak{M}$ in $\bar{E}$.

Assume now that a partially ordered sequence $\{f_{q_\alpha}\} \subset \mathfrak{M}$ converges weakly in E, i. e. there exists

$$\lim_\alpha f_{q_\alpha}(\varphi) \qquad (\varphi \in E).$$

Since Q is bicompact, the sequence $q_\alpha \subset Q$ possesses a directed condensation point $q_0 \in Q$. Then

$$\lim_\alpha f_{q_\alpha}(\varphi) = \lim_\alpha \varphi(q_\alpha) = \varphi(q_0) = f_{q_0}(\varphi) \qquad (\varphi \in E).$$

We see that $\mathfrak{M}$ is closed in ε_*.

Now put

$$F\left(\sum_1^n c_k f_{q_k}\right) = \sum_1^n c_k \varphi_0(q_k).$$

It is evident that this equality determines uniquely a linear functional on the linear hull $L(\mathfrak{M})$ over $\mathfrak{M}$. The functional F is additive and homogenous in $L(\mathfrak{M})$, and, as is easily seen,

$$\left|F\left(\sum_1^n c_k f_{q_k}\right)\right| \leqslant \left\|\sum_1^n c_k f_{q_k}\right\|.$$

Consequently, F is a linear functional in the space $L(\mathfrak{M}) \subset \bar{E}$. It can be extended on the whole space $\bar{E}$ so that its norm rests invariant. The continuity of $F(f)$ on the set $\mathfrak{M} \subset \varepsilon_*$ can be easily established. Applying theorem A, we obtain that for an element $\varphi_\varepsilon \in E$

$$|F(f) - f(\varphi_\varepsilon)| \leqslant \varepsilon \qquad (f \in \mathfrak{M}),$$

where ε is an arbitrarily chosen positive number. Consequently

$$|\varphi_0(q) - \varphi_\varepsilon(q)| \leqslant \varepsilon \qquad (q \in Q)$$

whence $\varphi_0 \in E$.

We have established that Theorem C is a corollary of Theorem A and, consequently, the latter is not true in general.

Active Red Army.

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THEORY OF ELASTICITY

ON AN ERROR IN A HYPOTHESIS OF THE THEORY OF SHELLS

By V. V. NOVOJILOV

(Communicated by B. G. Galerkin, Member of the Academy, 24.VII.1942)

1. Problems connected with the calculation of shells are conveniently dealt with by means of a triorthogonal system of coordinates (α and β are the main curvilinear coordinates of the middle surface of the shell, and z is the distance to this surface, measured towards its convex side along the normal).

Referring to these coordinates the equilibrium equations of the theory of elasticity and integrating them with respect to z over the interval from $-t/2$ to $+t/2$, that is all over the thickness of the shell, we have

$$\left.\begin{aligned}
&\frac{1}{AB}\left\{\frac{\partial BT_1}{\partial\alpha}+\frac{\partial AS_2}{\partial\beta}+\frac{\partial A}{\partial\beta}S_1-\frac{\partial B}{\partial\alpha}T_2\right\}+\frac{N_1}{R_1}+\overline{X}=0,\\
&\frac{1}{AB}\left\{\frac{\partial BS_1}{\partial\alpha}+\frac{\partial AT_2}{\partial\beta}-\frac{\partial A}{\partial\beta}T_1+\frac{\partial B}{\partial\alpha}S_2\right\}+\frac{N_2}{R_2}+\overline{Y}=1,\\
&\frac{1}{AB}\left\{\frac{\partial BN_1}{\partial\alpha}+\frac{\partial AN_2}{\partial\beta}\right\}-T_1/R_1-T_2/R_2+\overline{Z}=0.
\end{aligned}\right\} \tag{1}$$

Here A and B are Lamé parameters of the middle surface; R_1 and R_2 are the main radii of curvature.

$T_1, T_2, S_1, S_2, N_1, N_2$ are internal stresses acting within the shell.

$$\left.\begin{aligned}
T_1&=\int_{-t/2}^{+t/2}(1+z/R_2)\,\hat{\alpha\alpha}\,dz, \quad T_2=\int_{-t/2}^{+t/2}(1+z/R_1)\,\hat{\beta\beta}\,dz\\
S_1&=\int_{-t/2}^{+t/2}(1+z/R_2)\,\hat{\alpha\beta}\,dz, \quad S_2=\int_{-t/2}^{+t/2}(1+z/R_1)\,\hat{\alpha\beta}\,dz
\end{aligned}\right\} \tag{2}$$

$$N_1=\int_{-t/2}^{+t/2}(1+z/R_2)\,\hat{\alpha z}\,dz, \quad N_2=\int_{-t/2}^{+t/2}(1+z/R_1)\,\hat{\beta z}\,dz. \tag{3}$$

By $\overline{X}$, $\overline{Y}$ and $\overline{Z}$ we denote the given functions of the external load. For instance,

$$\overline{X}=(1+t/2R_1)(1+t/2R_2)X_1-(1-t/2R_1)(1-t/2R_2)X_2+\\+\int_{-t/2}^{+t/2}\rho(1+z/R_1)(1+z/R_2)F_\alpha\,dz, \tag{4}$$

where X_1 and X_2 are values of the stress $\hat{\alpha z}$ for $z=-t/2$ and $z=+t/2$. F_α is the component of the mass force in the direction α.

The formulae for $\overline{Y}$ and $\overline{Z}$ can be written down correspondingly.

The system of equations (1) is a mathematical expression of the conditions under which the principal vector of all internal and external stresses acting on a small element, mentally cut out of the shell, is equal to zero. Multiplying, further, by z two of the three equations of the theory of elasticity (referred to the coordinates α, β and z) and again integrating them with respect to z within the same range, we obtain two more equations:

$$\left.\begin{aligned} \frac{1}{AB}\left\{\frac{\partial BM_1}{\partial\alpha}+\frac{\partial AH_2}{\partial\beta}+\frac{\partial A}{\partial\beta}H_1-\frac{\partial B}{\partial\alpha}M_2\right\}-N_1-\mathfrak{M}_1=0 \\ \frac{1}{AB}\left\{\frac{\partial BH_1}{\partial\alpha}+\frac{\partial AM_2}{\partial\beta}-\frac{\partial A}{\partial\beta}M_1+\frac{\partial B}{\partial\alpha}H_2\right\}-N_2+\mathfrak{M}_2=0 \end{aligned}\right\} \tag{5}$$

Here M_1, M_2, H_1, H_2 are internal moments bending and twisting the shell.

$$\left.\begin{aligned} M_1=\int_{-t/2}^{+t/2} z\,(1+z/R_2)\,\hat{\alpha\alpha}\,dz,\quad M_2=\int_{-t/2}^{+t/2} z\,(1+z/R_1)\,\hat{\beta\beta}\,dz \\ H_1=\int_{-t/2}^{+t/2} z\,(1+z/R_2)\,\hat{\alpha\beta}\,dz,\quad H_2=\int_{-t/2}^{+t/2} z\,(1+z/R_1)\,\hat{\alpha\beta}\,dz \end{aligned}\right\} \tag{6}$$

and $\mathfrak{M}_1$, $\mathfrak{M}_2$ are known functions of the external load.

For instance

$$\mathfrak{M}_1=t/2\,[(1+t/2R_1)\,(1+t/2R_2)\,X_1+(1-t/2R_1)\,(1-t/2R_2)\,X_2]+ \\ +\int_{-t/2}^{+t/2}\rho z\,(1+z/R_1)\,(1+z/R_2)\,F_\alpha\,d^*. \tag{7}$$

The meaning of equations (5) is that the principal moment of all forces acting on an element of the shell has its tangential component to the middle surface equal to zero.

Taken together, the equations (1) and (5) present a mathematical expression of the necessary and sufficient condition for an elementary portion of the shell to be in elastic equilibrium (the sixth condition of statics, that the principal moment of all forces has its normal component to the middle surface equal to zero, can be shown to be always fulfilled identically).

2. In order to relate the stresses and moments to the displacements of the middle surface, Kirchhoff's hypotheses are generally used which may be reduced to the following assumptions:

a) In determining the deformations $e_{\alpha\alpha}$, $e_{\beta\beta}$, $e_{\alpha\beta}$, the deformations $e_{\alpha z}$, $e_{\beta z}$, e_{zz} can be taken to be zero.

b) The stresses $\hat{zz}$ can be neglected by comparison to the other stresses.

On the assumption (a) we obtain the following formulae for the deformations $e_{\alpha\alpha}$, $e_{\beta\beta}$, $e_{\alpha\beta}$ of an arbitrary point of the shell.

$$\left.\begin{aligned} e_{\alpha\alpha}=\frac{1}{1+z/R_1}(\varepsilon_1+z\varkappa_1),\quad e_{\beta\beta}=\frac{1}{1+z/R_2}(\varepsilon_2+z\varkappa_2) \\ e_{\alpha\beta}=\frac{1}{1+z/R_1}(\omega_1+2z\tau_1)+\frac{1}{1+z/R_2}(\omega_2+2z\tau_2)= \\ =\frac{1}{(1+z/R_1)(1+z/R_2)}\left\{\left(1-\frac{z^2}{R_1R_2}\right)\omega+\left[1-\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\frac{z}{2}\right]2z\tau\right\} \end{aligned}\right\} \tag{8}$$

where

$$\left.\begin{aligned}
\varepsilon_1 &= \frac{1}{A}\frac{\partial u}{\partial \alpha} + \frac{1}{AB}\frac{\partial A}{\partial \beta}v + \frac{w}{R_1}, & \varkappa_1 &= \frac{1}{A}\frac{\partial \vartheta}{\partial \alpha} + \frac{1}{AB}\frac{\partial A}{\partial \beta}\psi \\
\varepsilon_2 &= \frac{1}{B}\frac{\partial v}{\partial \beta} + \frac{1}{AB}\frac{\partial B}{\partial \alpha}u + \frac{w}{R_2}, & \varkappa_2 &= \frac{1}{B}\frac{\partial \psi}{\partial \beta} + \frac{1}{AB}\frac{\partial B}{\partial \alpha}\vartheta \\
\omega_1 &= \frac{1}{A}\frac{\partial v}{\partial \alpha} - \frac{1}{AB}\frac{\partial A}{\partial \beta}u, & 2\tau_1 &= \frac{1}{A}\frac{\partial \psi}{\partial \alpha} - \frac{1}{AB}\frac{\partial A}{\partial \beta}\vartheta \\
\omega_2 &= \frac{1}{B}\frac{\partial u}{\partial \beta} - \frac{1}{AB}\frac{\partial B}{\partial \alpha}v, & 2\tau_2 &= \frac{1}{B}\frac{\partial \vartheta}{\partial \beta} - \frac{1}{AB}\frac{\partial B}{\partial \alpha}\psi \\
\omega &= \frac{A}{B}\frac{\partial}{\partial \beta}\left(\frac{u}{A}\right) + \frac{B}{A}\frac{\partial}{\partial \alpha}\left(\frac{v}{B}\right), & 2\tau &= 2\tau_1 + \frac{\omega_2}{R_1} = 2\tau_2 + \frac{\omega_1}{R_2} \\
\vartheta &= -\frac{1}{A}\frac{\partial w}{\partial \alpha} + \frac{u}{R_1}, & \psi &= -\frac{1}{B}\frac{\partial w}{\partial \beta} + \frac{v}{R_2}
\end{aligned}\right\} \qquad (9)$$

u, v, w are the displacements of the middle surface.

When (8) is substituted into (2) and (6) [while using Hooke's law and taking into account the assumption (b)], and the latter formulae are then introduced into (1) and (5), the result is a system of five differential equations in partial derivatives with five unknowns u, v, w, N_1 and N_2 which depend only upon α and β. It is to the solution of this system that the computation of the shell's strength is reduced.

3. Now, however, there arises the question as to the error involved in Kirchhoff's hypothesis. To evaluate this error is important, first, because this would permit us to establish the boundaries, within which the theories of shells based on these hypotheses are applicable; secondly, because we should thus be in a position to simplify the theory by lawfully discarding the small terms whose order of magnitude does not exceed the error involved in the initial hypothesis.

4. In this connexion let us express with a higher degree of precision the deformation formulae (8). In deriving these formulae we, instead of neglecting the deformations $e_{\alpha z}$, $e_{\beta z}$, e_{zz}, shall define them by the first three terms of their expansion into MacLoren's series of the powers of z:

$$\left.\begin{aligned}
\hat{\alpha z} &= a_0 + a_1 z + a_2 z^2/2, \\
\hat{\beta z} &= b_0 + b_1 z + b_2 z^2/2, \\
\hat{z z} &= c_0 + c_1 z + c_2 z^2/2.
\end{aligned}\right\} \qquad (10)$$

On submitting these expressions to boundary conditions, we shall have

$$\left.\begin{aligned}
\hat{\alpha z} &= \frac{X_1 + X_2}{2} + (X_1 - X_2)\, z/t + \frac{a_2}{2}(z^2 - t^2/4), \\
\hat{\beta z} &= \frac{Y_1 + Y_2}{2} + (Y_1 - Y_2)\, z/t + \frac{b_2}{2}(z^2 - t^2/4), \\
\hat{z z} &= \frac{Z_1 + Z_2}{2} + (Z_1 - Z_2)\, z/t + \frac{c_2}{2}(z^2 - t^2/4).
\end{aligned}\right\} \qquad (11)$$

Note that by virtue of (3) the coefficients a_2 and b_2 can always be expressed by N_1 and N_2. No new notions are therefore introduced by these coefficients into the theory of shells.

As for the coefficient c_2, it will be defined approximately by subjecting to examination the quantity:

$$Q = \int_{-t/2}^{+t/2} (1 + z/R_1)(1 + z/R_2)\, \hat{z z}\, dz \qquad (12)$$

(the area of the epure of stresses $\hat{z z}$) from which we require that it should take its value in accordance with the equilibrium equations of the theory of

elasticity. Multiplying one of the equations by z and integrating over the thickness gives us the expression

$$Q=\frac{1}{AB}\left\{\frac{\partial}{\partial\alpha}\left[B\int_{-t/2}^{+t/2} z\,(1+z/R_2)\,\hat{\alpha z}\,dz\right]+\frac{\partial}{\partial\beta}\left[A\int_{-t/2}^{+t/2} z\,(1+z/R_1)\,\hat{\beta z}\,dz\right]\right\}-$$
$$-\frac{M_1}{R_1}-\frac{M_2}{R_2}+\mathfrak{M}_3 \qquad (13)$$

$\mathfrak{M}_3$ being defined by formula (7) in which X_1, X_2 and F_α are to be substituted with Z_1, Z_2 and F_z, respectively.

Equation (13) connects Q with N_1, N_2, M_1, M_2 and with the given values.

Making use of (10) and (11) one obtains, in place of formulae (8), more accurate expressions for the deformations $e_{\alpha\alpha}$, $e_{\beta\beta}$, $e_{\alpha\beta}$ (in the series only the first three terms are retained).

$$\left.\begin{aligned}
e_{\alpha\alpha}&=\frac{1}{1+z/R_1}\left\{\varepsilon_1+\left(\varkappa_1+\varepsilon_1^\circ-\frac{\mu}{1-\mu}\frac{\Omega}{R_1}\right)z+\left[\varkappa_1^0+\varepsilon_1'-\frac{\mu}{1-\mu}\frac{\Omega^\circ}{R_1}+\right.\right.\\
&+\frac{\mu}{1-\mu}\left[\frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{1}{A}\frac{\partial\Omega}{\partial\alpha}\right)+\frac{1}{AB^2}\frac{\partial A}{\partial\beta}\frac{\partial\Omega}{\partial\beta}\right]-\frac{\mu}{1-\mu}\frac{1}{R_1}\left[\gamma_1+\varkappa_2-\frac{\varepsilon_1}{R_1}-\right.\\
&\left.\left.\left.-\frac{\varepsilon_2}{R_2}-\frac{\mu}{1-\mu}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\Omega\right]\right]\frac{z^2}{2}+\dots\right\},\\
e_{\beta\beta}&=\frac{1}{1+z/R_2}\left\{\varepsilon_2+\left(\varkappa_2+\varepsilon_2^\circ-\frac{\mu}{1-\mu}\frac{\Omega}{R_2}\right)z+\left[\varkappa_2^\circ+\varepsilon_2'-\right.\right.\\
&-\frac{\mu}{1-\mu}\frac{\Omega^\circ}{R_2}+\frac{\mu}{1-\mu}\left[\frac{1}{B}\frac{\partial}{\partial\beta}\left(\frac{1}{B}\frac{\partial\Omega}{\partial\beta}\right)+\frac{1}{BA^2}\frac{\partial B}{\partial\alpha}\frac{\partial\Omega}{\partial\alpha}\right]-\\
&\left.-\frac{\mu}{1-\mu}\frac{1}{R_2}\left[\varkappa_1+\varkappa_2-\frac{\varepsilon_1}{R_1}-\frac{\varepsilon_2}{R_2}-\frac{\mu}{1-\mu}\left(\frac{1}{R_1}+\frac{1}{R_2}\right)\Omega\right]\right]\frac{z^2}{2}+\dots\right\},\\
e_{\alpha\beta}&=\frac{1}{1+z/R_1}\left\{\omega_1+(2\tau_1-\omega_1^0)\,z+\left[2\tau_1^\circ+\omega_1'+\right.\right.\\
&+\frac{\mu}{1-\mu}\left[\frac{1}{A}\frac{\partial}{\partial\alpha}\left(\frac{1}{B}\frac{\partial\Omega}{\partial\beta}\right)-\frac{1}{BA^2}\frac{\partial A}{\partial\beta}\frac{\partial\Omega}{\partial\alpha}\right]\left.\right]\frac{z^2}{2}+\dots\left.\right\}+\\
&+\frac{1}{1+z/R_2}\left\{\omega_2+(2\tau_2-\omega_2^\circ)\,z+\left[2\tau_2^\circ+\omega_2'+\right.\right.\\
&+\frac{\mu}{1-\mu}\left[\frac{1}{B}\frac{\partial}{\partial\beta}\left(\frac{1}{A}\frac{\partial\Omega}{\partial\alpha}\right)-\frac{1}{AB^2}\frac{\partial B}{\partial\alpha}\frac{\partial\Omega}{\partial\beta}\right]\left.\right]\frac{z^2}{2}+\dots\left.\right\}.
\end{aligned}\right\} \qquad (14)$$

Here

$$\left.\begin{aligned}
a_0'&=\frac{a_0}{G}, \quad a_1'=\frac{a_1}{G}, \quad b_0'=\frac{b_0}{G}, \quad b_1'=\frac{b_1}{G},\\
c_0'&=\frac{1-2\mu}{2(1-\mu)}\frac{c_0}{G}, \quad c_1'=\frac{1-2\mu}{2(1-\mu)}\frac{c_1}{G},\\
\Omega&=\varepsilon_1+\varepsilon_2.
\end{aligned}\right\} \qquad (15)$$

ϑ°, ψ°, $\varkappa_1^\circ$, ... are obtained from the formulae for the respective quantities without the zero index, when u, v and w are replaced by a_0', b_0' and c_0', respectively. Accordingly, ϑ', ψ', $\varkappa_1'$, ... are obtained from the same formulae by the substitution of u, v and w for a_1', b_1' and c_1'.

In the formulae (14) we see a number of correction terms which depend upon Ω, N_1, N_2 and Q and are not contained in (8). The significance of these corrections can be evaluated by determining Ω, N_1, N_2 and Q in first approximation, that is proceeding from the theory based on Kirchhoff's hypothesis.

5. Many problems of this theory have at present received their solutions, and so it may be easily revealed on particular examples in what measure the formulae (14) differ from formulae (8).

Upon these results it may be established that, generally speaking, the Kirchhoff's hypotheses in the theory of shells involve an error whose order

of magnitude is t/R compared to unity, R being the least of the curvature radii of the shell.

From this it follows that

a) Kirchhoff's hypotheses furnish a reliable foundation to a theory of thin shells, but are not fitted for the calculation of thick or even moderately thick shells [expression used by B. G. Galerkin (1)].

b) In the formulae of a theory of shells based upon Kirchhoff's hypotheses all terms of order t/R must be neglected in comparison with unity.

c) Although numerous authors (2-4 and oth.), while taking Kirchhoff's hypothesis as a basis, are bent upon retaining in their formulae the small terms of the order mentioned above, this is hardly consistent from the mathematical point of view, seeing that plainly superfluous terms are thereby introduced into the theory, which contribute in no way to the refining of the problem.

6. In accordance with what has been stated here the quantities z/R_1 and z/R_2 in the formulae (2), (6) and (8) are neglected by us in comparison to unity. This leads to the following formulae which relate the elastic stresses and moments to the deformations of the middle surface

$$\left.\begin{aligned} T_1 &= \frac{Et}{1-\mu^2}(\varepsilon_1+\mu\varepsilon_2), & M_1 &= D(\varkappa_1+\mu\varkappa_2), \\ T_2 &= \frac{Et}{1-\mu^2}(\varepsilon_2+\mu\varepsilon_1), & M_2 &= D(\varkappa_2+\mu\varkappa_1), \\ S_1 &= S_2 = \frac{Et}{2(1+\mu)}\omega, & H_1 &= H_2 = D(1-\mu)\tau. \end{aligned}\right\} \qquad (16)$$

These expressions are no novelty in the theory of shells. Many authors have used them and considered them to be sufficiently accurate. But what appears to be new in this communication, is the assertion—and an important one in point of principle,—that in a theory of shells resting upon Kirchhoff's hypotheses the formulae (16) present the only consistent way of establishing the relation between the stresses, moments and the deformations of the middle surface. Any attempt at making these formulae more accurate implies the necessity of renouncing Kirchhoff's hypothesis and building up instead a more precise theory. None of the available varieties of formulae (16), which include small terms of order t/R, can possibly be regarded as consistent from the mathematical point of view.

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AERODYNAMICS

A NEW METHOD FOR CALCULATING THE TURBULENT BOUNDARY LAYER AND DETERMINING THE SEPARATION POINT

By L. E. KALIKHMAN

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Considerable modifications are introduced by the boundary layer and wake in the picture which the streamlining of a profile presents on the theory of ideal fluid. The existence of the boundary layer accounts for the appearance of the profile drag; the separation of the turbulent layer is the main factor in decreasing the lift at great angles of incidence. Aside from theoretical interest, the investigation of these phenomena is of importance from the practical standpoint, and becomes more and more so in connexion with the increase of speed in aviation.

The averaged equation of the plane stationary boundary layer in an incompressible fluid can be brought to the form

$$\frac{1}{\bar{U}R_e}\frac{dR_\vartheta}{d\bar{s}} - R_\vartheta\omega^2\,(H+1) = \frac{1}{\zeta^2}\,, \tag{1}$$

where

$$R_\vartheta = \bar{U}\bar{\vartheta}R_e, \quad \omega^2 = -\frac{1}{\bar{U}^3R_e}\frac{d\bar{U}}{d\bar{s}}\,, \quad \zeta = \sqrt{\frac{\rho\overline{U^2}}{\tau_0}},$$

$$R_e = \frac{Vb}{\nu}\,, \quad \bar{U} = \frac{U}{V}\,, \quad \bar{\vartheta} = \frac{\vartheta}{b}\,, \quad \bar{s} = \frac{s}{b}$$

V, the velocity of the undisturbed flow; s, the length of the arc measured from the critical to a given point; b, the chord of the profile; ρ, density; ν, kinematic viscosity.

The velocity at the border of the layer U and its derivative are considered to be known functions of s. The quantity H is a practically constant value along the major portion of the length of the turbulent layer. The calculation of the turbulent boundary layer is here reduced to integrating the equation (1), while making use of this or that law of resistance, which is a supplementary ratio between momentum thickness ϑ and friction stress on the wall τ_0.

In 1931—1932 A. Buri [1] and H. Müller obtained a solution of this problem in quadratures which was based on a power law of resistance. However, this law suffering from essential drawbacks (deviation from the experiment, dependence of the exponent upon R_e, etc.), one was compelled to pass to the more accurate logarithmic laws of velocity distribution and resistance. But then the equation (1) had each time to be integrated anew with the use of most laborious numerical methods.

Below, the equation of the plane turbulent boundary layer is given in its final form for logarithmic law of resistance*.

* A similar solution is obtained for a flow with axial symmetry.

Adopting T. Kármán's logarithmic law of resistance, in the form suggested by H. Squire and A. Joung (2)

$$R_\vartheta = C_1 e^{\varkappa\zeta} \tag{2}$$

and performing the substitution of variables

$$z = R_\vartheta \zeta^2 \tag{3}$$

we can reduce the equation (1) to a linear equation with respect to z:

$$\frac{dz}{d\bar{s}} + K(H+1)\frac{\frac{d\bar{U}}{d\bar{s}}}{\bar{U}} z = K\bar{U}R_e. \tag{4}$$

The quantity $K = 1 + \frac{2}{\varkappa\zeta}$ is liable to no substantial variation in a turbulent boundary layer, and it may be assumed with a high degree of accuracy that

$$K(H+1) = \text{const}. \tag{5}$$

The general integral of the linear equation (4) has the form

$$z = \bar{U}^{-K(H+1)}\left[C + R_e \int K\bar{U}^{K(H+1)+1}\, d\bar{s}\right]. \tag{6}$$

Putting the constant (5) equal to 3 (in the region of minimum pressure $K \cong 1.25$, $H \cong 1.4$; in the separation region $K = 1$, H is of order 2) and taking 1.2 for the mean value of K under the integral sign, we obtain the following formula in quadratures:

$$z = \frac{C + 1.2R_e \int_{\bar{s}_t}^{\bar{s}} \bar{U}^4\, d\bar{s}}{\bar{U}^3}. \tag{7}$$

The method of calculation is remarkable for its simplicity. From a given distribution of pressure $\bar{P}$ we get $\bar{U} = \sqrt{1-\bar{P}}$. We plot two auxiliary curves bringing out the variation of $\lg z$ with ζ and of $\lg R_\vartheta$ with ζ according to formula (3) and (2), respectively*. R_ϑ at the beginning of the turbulent region ($\bar{s} = \bar{s}_t$) is taken to have the same value which it has at the end of the laminar area. For computing the latter and for ascertaining the position of the transition point we may avail ourselves of a method suggested previously (3). From $\lg R_{\vartheta_t}$ we find ζ_t, $\lg z_t$, z_t, $C = z_t\bar{U}_t^3$. For $\bar{s}_t = 0$ we have $C = 0$; z is computed as a function of $\bar{s}$; ζ and R_ϑ are found from the plot.

The values of ζ serve to determine the distribution of stress friction; from the value of R_ϑ on the trailing edge the profile drag may be found, and the distribution of R_ϑ will give us the separation point.

The assumptions upon which the obtained solution is based, in particular, that the pressure gradient has no influence on the velocity profile, were subjected to a careful investigation. It was found out that in spite of the fact that the influence of the pressure gradient on the resistance law and on H is considerable, the effect of this influence upon R_ϑ is negligibly small. Formula (7), though extremely simple, is highly accurate in defining R_ϑ (of an order of 3 to 4%) not only for small, but even for critical angles of incidence.

* It is well to put $C_1 = 0.3126$ and $\varkappa = 0.375$ in accordance with the condition that the resistance of the plate obeys Schultz-Grunov's formula.

The problem concerning the separation of the turbulent layer is not solved by merely solving the equation (1), but there is yet to be laid down a special criterion.

Qualitatively the separation of the turbulent layer is analogous to the separation of the laminar layer; from a certain section onward the flow leaves the wall, and a zone of back flow is formed on the profile.

In the neighbourhood of the separation point the derivative of the velocity u with respect to the normal y on the wall changes its sign; hence the separation point is characterized by the condition

$$\left.\frac{\partial u}{\partial y}\right|_{y=0} = 0. \tag{8}$$

From (8) and from the equation of the boundary layer it follows that the profile of the friction stress at the point of separation should for $\eta = \frac{y}{\delta} = 0$ satisfy the following conditions [cf. (4) and also (1)]:

$$\bar{\tau} = \frac{\tau}{\rho U^2} = 0, \quad \frac{\partial \bar{\tau}}{\partial \eta} = R_\delta \omega^2, \quad \frac{\partial^2 \bar{\tau}}{\partial \eta^2} = 0, \quad \frac{\partial^3 \bar{\tau}}{\partial \eta^3} = 0 \tag{9}$$

$\left(\delta$, thickness of the boundary layer; $R_\delta = \frac{U\delta}{\nu}\right)$. By the definition of the boundary layer we have, for $\eta = 1$,

$$\bar{\tau} = 0, \quad \frac{\partial \bar{\tau}}{\partial \eta} = 0. \tag{9'}$$

Assuming that the profile of the friction stress at the point of separation can be expressed as a polynomial, we find

$$\bar{\tau} = R_\delta \omega^2 f(\eta). \tag{10}$$

By means of Prandtl's formula (5)

$$\tau = \rho l^2 \left(\frac{du}{dy}\right)^2 \tag{11}$$

we obtain the turbulent profile of velocities at the point of separation. As the section of the separation is the beginning of an attending stream, in which, like in a free stream, the dimensionless path of mixing can be regarded as constant, we may take $\frac{l}{\delta} = c$.

Then (10) and (11) give

$$\frac{u}{U} = 1 - \frac{1}{c}\sqrt{R_\delta \omega^2} \int_\eta^1 \sqrt{f(\eta)}\, d\eta. \tag{12}$$

Conditions (9) show that in the laminar sublayer we can put

$$\bar{\tau} = R_\delta \omega^2 \eta. \tag{13}$$

Hence

$$\frac{u}{U} = \frac{R_\delta^2 \omega^2 \eta^2}{2}. \tag{14}$$

Let the boundary of the laminar layer be defined, as always, by assuming the existence of Reynold's critical number (3,4):

$$\frac{u_e \delta_e}{\nu} = \alpha^2. \tag{15}$$

The substitution of (14) in (15) gives

$$\eta_0 = \frac{1}{R_\delta} \sqrt[3]{\frac{2\alpha^2}{\omega^2}}. \tag{16}$$

By equating the right-hand sides of the expressions (12) and (14), substituting (16) and discarding the small term, we get

$$\left(a=\int_0^1 \sqrt{f(\eta)}\,d\eta,\quad A=\sqrt[3]{\frac{a^4}{2}}\right)$$

$$\sqrt{R_\delta\omega^2}=\frac{c}{a}\,(1-A\sqrt[3]{\omega^2}). \tag{17}$$

After (12) is substituted in the formula

$$\frac{\vartheta}{\delta}=\int_0^1 \frac{u}{U}\left(1-\frac{u}{U}\right)d\eta, \tag{18}$$

we have $\left(M=\int_0^1\int_\eta^1 \sqrt{f(\eta)}\,d\eta d\eta,\ P=\int_0^1\left[\int_\eta^1 \sqrt{f(\eta)}\,d\eta\right]^2 d\eta\right)$:

$$\frac{\vartheta}{\delta}=\frac{M}{c}\sqrt{R_\delta\omega^2}-\frac{P}{c^2}R_\delta\omega^2. \tag{19}$$

From (17) and (19) we get

$$R_\delta\omega^2=\frac{c^2}{a^2}\left[\frac{M}{a}\,(1-A\sqrt[3]{\omega^2})-\frac{P}{a^2}\,(1-A\sqrt[3]{\omega^2})^2\right](1-A\sqrt[3]{\omega^2})^2. \tag{20}$$

The available experimental data do not permit as yet to determine the value of the experimental constant c in a reliable manner. There is also a certain degree of arbitrariness in the definition of the constants a, M and P*.

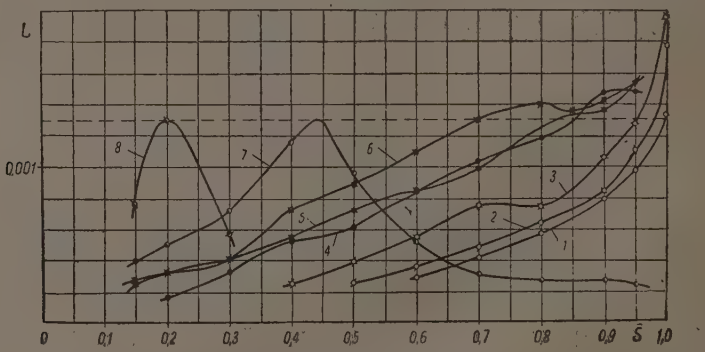

Fig. 1. Distribution of parameter L across the profile $B=12\%$, $R_e=1.1\times10^6$. *1*—$C_y=0.300$; *2*—$C_y=0.400$; *3*—$C_y=0.600$; *4*—$C_y=0.800$; *5*—$C_y=0.930$; *6*—$C_y=1.000$; *7*—$C_y=0.040$; *8*—$C_y=0.845$.

The number of constants can be reduced from 4 to 1, if the following fact, ascertained by experiment, is taken into consideration; the velocity profiles of the turbulent layer in the region of separation have a distinctly linear character within the major part of the layer's thickness. This means that we can put $\sqrt{f(\eta)}=k$. Equation (20) takes now the shape

$$R_\delta\omega^2=\left(\frac{c}{k}\right)^2[0.5\,(1-A\sqrt[3]{\omega^2})-0.333\,(1-A\sqrt[3]{\omega^2})^2]\,(1-A\sqrt[3]{\omega^2})^2. \tag{21}$$

It is interesting to note that separation in the case of turbulent or laminar layer is defined by the same parameters R_ϑ and ω. Indeed, it follows from the condition

$$\lambda=\overline{\delta}^2 R_e\frac{d\overline{U}}{d\overline{s}}=-12,$$

* The constant a has its usual value of 11.2.

obtained by the method of Kármán-Pohlhausen ([6], [7]) that at the separation point of the laminar layer

$$R_\vartheta = \frac{0.395}{\omega}.$$

The condition for the separation of the turbulent layer (21) may easily be reduced to a form more convenient for use. Within the limits of such values of ω as are of a practical interest, equation (21) can be represented in the form

$$R_\vartheta \omega^2 R_\vartheta^{-0.08} = \text{const}. \quad (22)$$

The single empirical constant of the problem has been found, according to the available experimental material, to have a value of 0.0013. Thus, for ascertaining the position of the separation point it will suffice to compute the distribution of the parameter $L = R_\vartheta \omega^2 R_\vartheta^{-0.08}$ and to find the value of the coordinate $\bar{s}$ corresponding to $L = 0.0013$.

There was investigated the position of the separation point on the upper surface of the profiles $B = 12\%$ and NACA-230 $= 17\%$.

The distribution of pressure was taken from ([8]). For every $C_y \geqslant 0.8$ the boundary layer was supposed to be turbulent throughout. For $C_y < 0.8$ the position of the point of transition was taken in accordance with the experimental data at hand. The results are represented in Figs. 1—3.

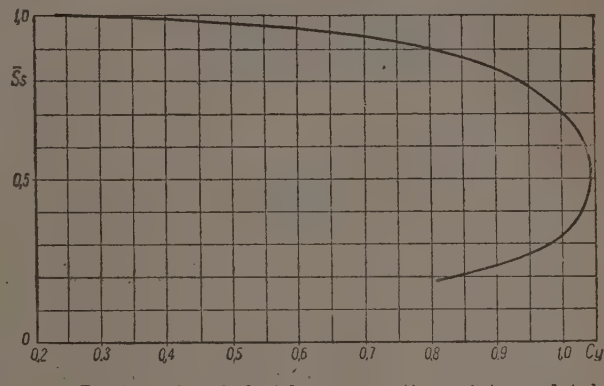

Fig. 2. Position of turbulent layer separation point as related to C_y (profile $B = 12\%$; $R_e = 1.1 \times 10^6$).

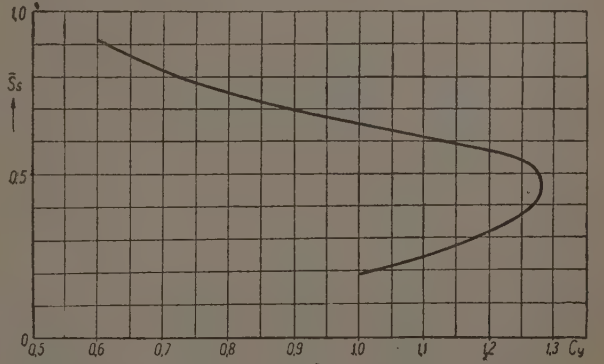

Fig. 3. Same as in Fig. 2 (profile NACA-230, $C = 17\%$, $R_e = 12 \times 10^6$).

By means of the substitution of variables pointed out by A. A. Dorodnitzyn ([9]), the formula in quadratures (7) and the separation criterion (22) can easily be generalized to the case of compressible fluid.

Central Aero-Hydrodynamical Institute.

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PHYSICS

THE RELATION BETWEEN CHEMICAL COMPOSITION AND GLASS TRANSMISSION IN THE INFRARED

By J. I. GERLOVIN

(Communicated by S. I. Wawilow, Member of the Academy, 12.X.1942)

Only two investigations have been made concerning the dependence of glass transmission in the infrared on the main glass-forming oxides. One of them is due to Dreisch(1), the other to Schmidt, Gelhoft and Thomas (2). Dreisch has studied the transmission of borate glasses of a rather complicated chemical composition; hence it is difficult to determine in that case the part played by the additional oxides and their influence on the glass transmission. Schmidt, Gelhoft and Thomas have chosen more simple systems—ternary ones. However, they have not been able to detect feeble absorption bands, as the work was done with a wide open slit of the monochromator. Binary systems, which show more clearly the dependence of glass transmission on individual components, they did not investigate at all. We chose some simple systems consisting of glass-forming oxides most often used in glass-making; with these systems we made an attempt of a systematic investigation of glass transmission in dependence on their chemical composition.

54 various types of glass have been studied. 30 of them were borate glasses, 20 silicate glasses and 4 phosphate glasses. All glasses have been made by R. I. Bresker and M. S. Heinrich (State Optical Institute). In the case of borate and silicate glasses binary systems have been measured, as well as ternary ones. The binary borate and silicate systems studied contained oxides of light and heavy metals. The only types of phosphate systems taken were binary ones containing oxides of heavy metals. The ternary systems consisted of oxides $5B_2O_3K_2O$ and $5SiO_2K_2O$ with the addition of different oxides of metals. Besides these binary and ternary systems, the oxides SiO_2 and B_2O_3 in vitreous state have been investigated. The latter oxide needs special mentioning. The anhydrous boron oxide in the air soon becomes turbid and is destroyed; therefore it is extremely difficult to measure its transmission. Hence a glass consisting of 99.5% B_2O_3 and 0.5% PbO has been taken. It is likely that the absorption bands of this glass coincide with the absorption bands of boric anhydride.

Among all borate glasses the boric anhydride has the least transmission in the infrared. It shows absorption bands at 2.2 μ, 2.4 μ and 2.85 μ (Fig. 1). A plate of this glass 1 mm thick at 3 μ practically transmits nothing at all. The absorption band at 2.85 μ is observed for all borate glasses; as to the bands at 2.2 μ and 2.4 μ, they are observed only for lighter glasses. For all borate glasses is also observed a band at 3.7 μ. With the increase of the percentage of additional oxides the transmission of borate glass in the infrared increases; the more heavy the oxide, the more marked the increase (Fig. 2). Thus, among the 30 types of borate glasses investigated the greatest transmission have lead-borate glasses, the smallest,—

apart from boric anhydride itself,—the lithium-borate glasses. In this way for all borate glasses an increase of transmission in the infrared with the increase of the specific gravity of metal oxides takes place. The increase of the percentage of additional oxides not only increases the transmission in the infrared, but also makes the absorption maxima, typical for boric anhydride, less pronounced. All absorption maxima show a shift 0.1—0.15 μ with respect to their position for boric anhydride.

All glasses with a high content of boron in the air soon become turbid, which results in a decrease of their transmission in the near infrared. This decrease may be explained by scattering rather than by absorption. In the region of longer waves the scattering is not essential.

In the case of phosphate glasses four types of binary system have been studied, two of them belonging to borium-phosphate glasses and two to lead-phosphate glasses. They all show

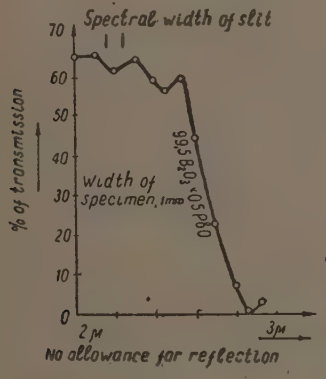

Fig. 1.

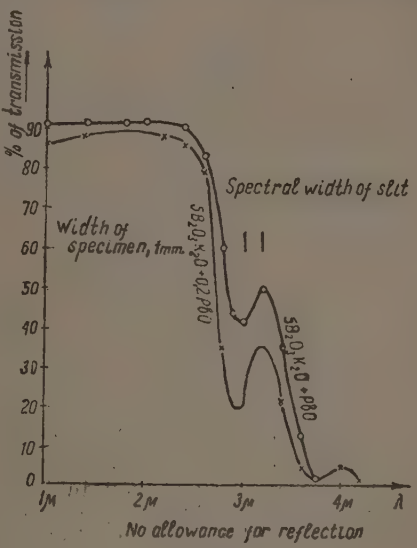

Fig. 2.

a similar form of the absorption curve (Fig. 3). At 2.2 μ an appreciable absorption begins, at 3.5 μ the plates measured (1 and 2 mm thick) practically do not transmit anything. The increase of the percentage of the metal oxide increases the transmission, but in a lesser extent than for borate glasses. In the near infrared phosphate glasses do not show any typical absorption maxima; in the region of greater wave-lengths we did not study them.

Silicate glasses have a greater durability and therefore are most often used.

The binary and ternary silicate systems studied by us are only slightly differing from ordinary commercial glasses. The binary systems studied were soda-silicate, potash-silicate and lead-silicate glasses. For each system the ratio of the contents of silicate and of metal oxide has been varied. Similar to the case of borate systems, the greatest transmission is shown by lead-silicate glasses (Fig. 4). The substitution of lead oxide by lighter metal oxides decreases the glass transmission in the infrared. The increase of the percentage of the oxide of the alkali metal in a binary silicate system (when the content is not less than 30%) strongly decreases the glass transmission in the infrared. This seems to be connected with the devitrification of these types of glass.

In the region from 4 μ to 5 μ lead-silicate glasses transmit better than quartz glass. In the region 2.8 μ—3.8 μ lead-silicate glasses have a broad, but a feeble absorption band.

All the three-component silicate systems investigated have nearly the same absorption curves. In contrast with the fused quartz that has a sharp absorption

maximum at 2.7 μ, for all silicate glasses (binary and ternary) this maximum is absent. Instead of it some silicate glasses show a feeble absorption maximum at 2.3—2.4 μ. The absorption band of fused quartz at 3.7 μ, that is feebler than the band at 2.7 μ, is observed for all glasses, but shows a shift 0.1—0.2 μ towards shorter wave-lengths.

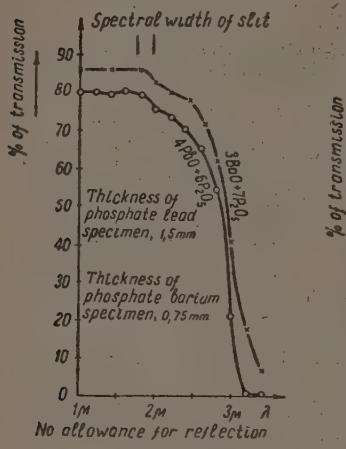

Fig. 3.

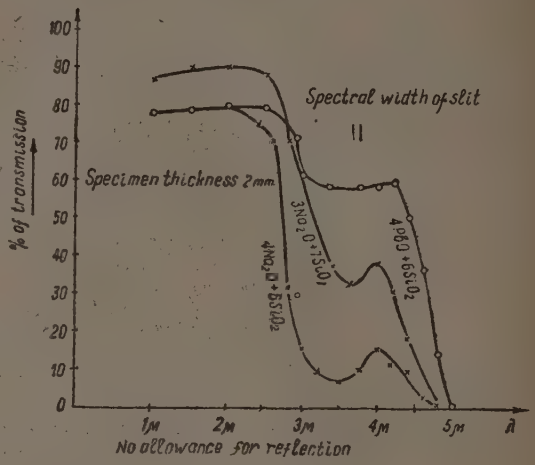

Fig. 4.

On the ground of analysis of infrared absorption spectra investigated by us the following conclusions may be done:

1. The greatest transmission in the infrared is shown by silicate systems; among the silicate systems—by lead-silicate ones.

2. The boric and phosphoric anhydrides increase the absorption in any system.

In conclusion, I am glad to express my thanks to Prof. M. L. Weingerov for the direction of this work and for valuable suggestions made.

Laboratory of Infrared Rays,
State Optical Institute.

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PHYSICS

To my dear Mother

ON THE THEORY OF MESONS

By K. NIKOLSKY

(Communicated by S. I. Wawilow, Member of the Academy, 29.VI.1942)

It is known that the action of the electromagnetic field on a relativistic quantum particle, described by the Dirac equation, is determined by a law, according to which the outer electromagnetic field enters into these equations. This law has a general and a very peculiar character, namely the presence of the outer electromagnetic field is taken account of in the equations by the transition from the usual partial derivatives $i\partial/\partial x_\alpha$ to the so-called covariant derivatives

$$D_\alpha = \left(\frac{1}{i}\frac{\partial}{\partial x_\alpha} + \frac{e}{\hbar c}\varphi_\alpha\right).$$

Thus, the action of the outer electromagnetic field on the particle is formulated on the same foundation as the inertia effects in Einstein's theory of gravitation, with the fundamental difference, however, that in the latter case we have the laws of parallel displacement of a vector or a tensor, while in the former quantum case we deal with an essentially new type of quantities — the spinors*.

Very recently Pauli (4) has treated the problem of a systematic classification of various possible types of equations of motion of elementary particles on the basis of this transformation laws, i. e. according to the irreducible representations of the Lorentz group.

We may consider the action of the outer meson field on the heavy particle, making use of a similar treatment of the problem. Indeed, the possibility of such a viewpoint becomes particularly clear, if we write the equations of motion of a nuclon in the form given by Möller (5):

$$\left\{\gamma^\alpha D_\alpha + \frac{\mu_0 c}{\hbar}\right\}\psi = 0 \text{ (summarizing from 1 to 4).} \qquad \text{(A)}$$

---

* If A^μ is a component of a vector, then, as is known, $\partial A^\mu = -\Gamma^\mu_{\nu\alpha} A^\alpha dx_\mu$, which may be expressed by the transition from the derivatives $\partial/\partial x_\alpha$ to the derivatives $\nabla_\nu = (\partial/\partial x_\nu + \Gamma^\mu_{\nu\alpha})$, where $\Gamma^\mu_{\nu\alpha}$ are the components of the affin connexion and characterize, as is known, the action of the gravitation field on the particle.

Let us remind that the theory or parallel displacement of a semi-vector (spinor ψ_ϱ) has been developed mainly in papers of theoretical physicists of Leningrad and also by Eddington (2) and Weyl (3) and was taken as the foundation of the customary quantum theory of relativistic particles.

Here are

$$D_\alpha = \frac{\partial}{\partial x_\alpha} - \frac{ig_1}{\hbar c}\vec{u}_\alpha\vec{\tau} + \frac{ig_2}{4k\hbar c}\vec{g}_{\mu\nu}\vec{\tau}[\gamma_\alpha, \gamma_\nu]\gamma_\alpha$$

$$\mu_0 = \frac{1+\vec{\tau}_3}{2}\mu_N^0 + \frac{1-\vec{\tau}_3}{2}\mu_P^0$$

where μ_N^0 and μ_P^0 are the ordinary rest masses of neutron and proton, and

$$\vec{\tau} = (\vec{\tau}_1, \vec{\tau}_2, \vec{\tau}_3); \quad [\vec{\tau}_3, \vec{\tau}_1] = 2i\vec{\tau}_2 \text{ etc.}$$

is the «isotopic spin-vector» chosen in such a way that the eigen values $+1$ and -1 of τ_3 refer to the neutron and proton states, respectively:

$$[\gamma_\mu, \gamma_\nu] = \gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu$$

is the commutator of γ_μ and γ_ν.

The variables γ_ν satisfy the commutation rules

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu} \qquad (\mu, \nu = 0, 1, 2, 3, 4).$$

The meson outer field is described by a four-vector $\vec{u}_\mu$ and an antisymmetrical tensor $\vec{g}_{\mu\nu}$.

The equations contain two universal constants g_1, g_2, which determine the couplings between the nuclons and the vector and pseudoscalar outer meson field, respectively.

The five quantities (x_α) are coordinates of a point in a Euclidian space R_5 of five dimensions. $x_4 = ict$ is the usual imaginary time variable, while x_1, x_2, x_3 are the ordinary Cartesian space coordinates. The real variable x_0 has no classical analogon, while the canonically conjugated momentum p_0 is directly connected with the proper mass of nuclon.

The nuclon equations of motion (A) are invariant under all rotations in a five-dimensional space R_5 (the rotations in R_5 represented by

$$x'_\mu = \sum_\nu a_{\mu\nu}x_\nu, \qquad \sum_\rho a_{\mu\rho}a_{\nu\rho} = \sum_\rho a_{\rho\mu}a_{\rho\nu} = \delta_{\mu\nu},$$

$$\Delta_5 = \det|a_{\mu\nu}| = +1$$

leave

$$\sum_{\mu=0}^{4} x_\mu^2 = \text{const}$$

invariant).

The group of rotations in R_5 induces a group of transformations of spinors ψ. The transformed spinor ψ' will be connected with ψ by the relation

$$\psi' = S\psi$$

where S is given by

$$S = 1 + \frac{1}{8}\sum_{\mu,\nu=0}^{4} e_{\mu\nu}[\gamma_\mu, \gamma_\nu]$$

for an infinitesimal transformation

$$x'_\mu = x_\mu + \sum_{\mu=0}^{4} e_{\mu\nu}x_\nu, \quad e_{\mu\nu} = -e_{\nu\mu}.$$

The investigation of the character of the affin connexion of the variety R_5 determining the parallel displacement of spinors ψ in R_5, offers considerable interest. In order to carry it out, we must look into the laws of transformations of the invariants bilinear in ψ and ψ^+ and of tensors of different rank*.

First of all, we may consider the action of the outer meson field on the heavy particle, making use of the conception of a parallel displacement of a five-dimensional spinor ψ_5.

Just as in the Dirac equations the first four terms express the infinitesimal variation of a four-dimensional spinor, so the combination of the first five terms in the nuclon equations can be regarded as the expression for the infinitesimal displacement of the five-dimensional spinor $\psi_5(x_0, x_1, x_2, x_3, x_4)$.

Thus, it is possible to develop a theory of representations of the transformations in R_5, similar to the theory of spinors of the Lorentz relativistic group, and to obtain systematization of possible types of particles, analogous to that performed by Pauli, i. e. a systematization according to irreducible representations of group transformations in R_5. Very interesting relationships take place between the groups considered, since the rotational group in R_5 is isomorphic with the stereographic projection of the group of conformal transformations in R_5, which fact is fundamental for understanding the experimental quantum electrodynamics ([6]).

We expect to consider this question in one of the notes soon to follow.

P. N. Lebedev Physical Institute.
Academy of Sciences of the USSR.

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* In the meson theory, the field and the sources are described by four-vector $\vec{u}_k$ and an antisymmetrical tensor $\vec{g}_{kl} = -\vec{g}_{lk}$, and four-vector $\vec{\mu}_k$ and an antisymmetrical tensor $\vec{S}_{kl} = -\vec{S}_{lk}$, respectively. The source densities are given by

$$\vec{\mu}_k = g_1 \psi^+ \vec{\tau} \gamma_k \psi, \quad \vec{S}_{kl} = \frac{g_2}{2k} \psi^+ \vec{\tau} [\gamma_k, \gamma_l] \psi.$$

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CHEMISTRY

FUSIBILITY DIAGRAM OF THE TERNARY SYSTEM SODIUM CHLORIDE-FLUORIDE-CHROMATE

By I. S. RASSONSKAYA and A. G. BERGMANN

(Communicated by V. G. Chlopin, Member of the Academy, 27.XI.1942)

This investigation of the system NaCl—NaF—Na_2CrO_4 has been one of the stages in a study carried out in the Laboratory of Molten Salts, Institute of General and Inorganic Chemistry, Academy of Sciences of the USSR. The object of the study was to find suitable baths for the thermal treatment of metals, carriers of heat and fluxes for welding and refining of metals.

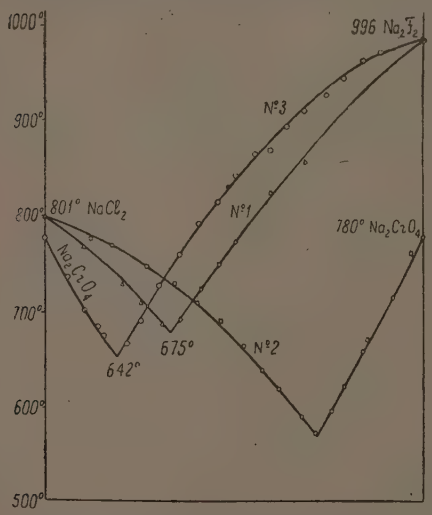

Fig. 1. Binary systems NaCl — NaF; Na_2Cl_2—Na_2CrO_4; Na_2F_2 — Na_2CrO_4.

The ternary system under consideration comprises three binary ones, namely NaCl—NaF, NaCl—Na_2CrO_4 and NaF—Na_2CrO_4. Some of the latter systems have been studied earlier; thus, the system NaCl—NaF has been investigated by Plato(1) and Wolters (2), and the system NaCl—Na_2CrO_4 has to some extent (in the range of 0 to 5.73 mol % Na_2CrO_4) been explored by Sackur (3) in connexion with the study of molecular depression in molten salts.

The authors have verified the system NaCl—NaF and studied to full extent the fusibility diagrams of the binary systems NaCl — Na_2CrO_4 and NaF—Na_2CrO_4 (Fig. 1), and also that of the ternary system NaCl—NaF—Na_2CrO_4 (Fig. 2). The determinations have been carried out by visual-polythermal method in a platinum crucible by means of a Pt—PtRh thermocouple (0.5 mm in diameter) and a Siemens-Halske mirror galvanometer.

According to our data, mainly concurrent with the results of the previous authors, the system NaCl—NaF is one with a simple eutectic having a composition 33.5 mol% of NaF and a melting point of 675° C.

The system NaCl—Na_2CrO_4 is also a simple one with eutectic point at 572°, to which there corresponds a composition of 78.5 mol % of Na_2CrO_4.

The system NaF—Na_2CrO_4 is one with a simple eutectic having a composition of 81.5 mol % of Na_2CrO_4 and a melting point of 642°.

Unlike an analogous system of potassium salts $KF—K_2CrO_4$, among which a compound $2KF \cdot K_2CrO_4$ has been discovered by E. P. Dergunov, the system $NaF—Na_2CrO_4$ has not been found to contain the respective compound. In the system $NaF—Na_2SO_4$ there is a compound $NaF \cdot Na_2SO_4$ melting at 781°.

In studying the ternary system six cross-sections have been examined, shown in Fig. 2 by dotted lines.

Since the component binary systems are systems with the simple eutectic, the liquidus surface of the ternary system is composed of 3 sharply demarca-

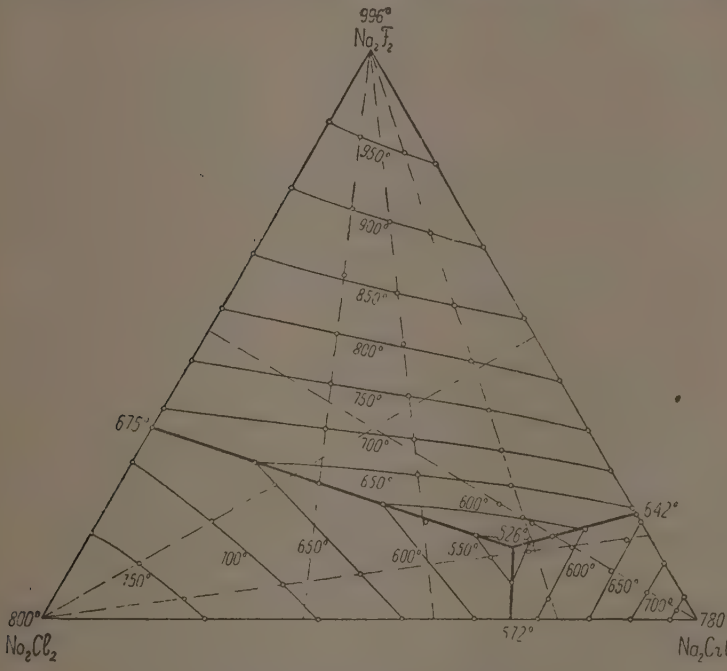

Fig. 2. Polytherm of ternary system Na || F, Cl, CrO_4.

ted crystallization areas. The crystallization area of NaF occupies the major part of this surface (61.4%), the next one is the crystallization area of NaCl (30.8%) and the last part is the area of Na_2CrO_4 (7.8%).

The ternary eutectic point has the composition 13 mol% of NaF, 21.5 mol% of NaCl and 65.5 mol% of Na_2CrO_4, or in per cent by weight: 7.68% NaF, 17.68% NaCl and 74.64% Na_2CrO_4, and corresponds to a temperature of 526° C.

At the same time a considerable depression of the melting points should be pointed out in the binary, as well as in the ternary systems.

The eutectic mixture is of interest as a salt bath for thermal treatment of metals and as a high temperature carrier of heat.

Laboratory of Molten Salts.
Institute of General and Inorganic Chemistry.
Academy of Sciences of the USSR.

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PHYSICAL CHEMISTRY

DAMPING OF DETONATION IN ELONGATED CHARGES ON INCREASED INITIATION

By A. F. BELAYEV

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The initiation of an explosive is understood in the most general sense to mean the process of stimulation of an explosion in it.

Very often, however, particularly in practice, the term initiation is applied to a narrower range of phenomena, namely, when the explosive decomposition is stimulated by an auxiliary charge of another explosive, the initiator.

Usually, a detonator serves as initiator. But sometimes a more powerful initiating impulse is resorted to. This is necessary when the substance is of little susceptibility and the detonator is either altogether incapable of stimulating the explosion or stimulates but incomplete explosive decomposition.

Thus, in the case of fused trotyl, which is a substance of poor susceptibility, an additional or intermediate initiator is often used, viz. a charge of pressed trotyl, exploded by a detonator.

The application of increased initiation is based on a vast accumulation of experimental material testifying to the fact that the more powerful initiating impulse stimulates, under identical conditions, a more complete explosive decomposition, greater detonation velocity, and the effect of explosion of the initiated charge is, eventually, stronger.

But there are exceptions to this general rule, as the present paper will show. Under certain conditions the more powerful initiating impulse may even lead to damping of detonation or to its weakening, whereas the less powerful one provides a stable expansion of detonation.

These data have been obtained by the author in common with E. V. Rdultovskaya during the study of the expansion and damping of detonation in elongated charges of ammonites of various composition, using the chronophotographic method. When a mixture of 97% of ammonium nitrate and 3% of trotyl was exploded in a glass tube, 40 to 45 cm long and 33 to 34 mm in the inner diameter, at a density of 0.8 to 0.9, a detonator serving for initiator, the result was a stable expansion of detonation through the full length of the cartridge at a little, but constant velocity of 1200 to 1300 m/sec. In the same mixture and under the same conditions, but with a more powerful initiator—an additional charge of pressed tetryl (5 g) or a charge of amitol 78/22 (60 to 70 g)—the detonation was damped in most cases. The photographs of the detonation, obtained by chronophotographic method, using a rapidly rotating drum, have shown that in the vicinity of the additional initiator the velocity of detonation in a 3% mixture is much higher than the constant velocity generated by a detonator. This increased velocity (1500—1800 m/sec.) is, however, unstable; it diminishes rapidly and the detona-

tion is damped. In some of the cases, when an additional initiator was used, the detonation was observed to expand through the full length of the cartridge. Nevertheless, an application of the chronophotographic method permits to discover that in this case, too, the expansion velocity of the detonation decreases continuously and at the end of the cartridge is only 1000 to 1100 m/sec., lower than the constant velocity which arises when the detonator alone is used for initiation. It may be expected that with a longer cartridge the damping of detonation with increased initiation would take place in every experiment. By the way, it should be pointed out here, that had we used instead of the chronophotographic method of measuring the detonation velocity any other method giving mean values of the velocity, we would have obtained greater values of the mean velocity in all the cases where the detonation expanded through the whole length of the cartridge in spite of additional initiator being used. The low velocity at the end of the cartridge (1000 to 1100 m/sec.) would have been abundantly compensated by the relatively high velocity (1500 to 1800 m/sec.) in the vicinity of the initiator. Therefore, we should have arrived in the end at a wrong conclusion that the application of the additional initiator increased the expansion velocity.

The chronophotographic method, giving the complete picture of the expansion of detonation, shows that under the given conditions the application of more powerful impulse actually leads to unstable damped detonation decomposition, sharply different in its general nature from the stable expansion stimulated by relatively weak impulse.

Experimental data at the author's disposal give no full idea as to the universality of the «super-initiation» effect.

It should be regarded, presumably, that this phenomenon lies within a narrow range, since no analogous fact has more or less clearly been stated in literature with a sole exception of a paper by Sherrick ([1]). In studying the detonation of ammonium nitrate stimulated by various initiators, Sherrick has found that under certain conditions relatively weak ammonite with a large content of ammonium nitrate provides better detonation of the latter than such compounds as tetryl, picric acid, etc. This fact, very similar to that discovered by the author, belongs apparently to the same category of phenomena and is due to the same cause which is not clear for the present.

The phenomenon of «super-initiation», definitely established for the mixture of 3% of trotyl and 97% of ammonium nitrate, has been registered by the author for a number of other mixtures.

It has also been pointed out that under certain conditions, when the detonator is incapable of providing stable detonation, the latter may be brought about by a weaker initiator, viz. a piece of detonating fuse inserted into the substance 1 to 2 mm deep.

To put an example, the detonation of trotyl of a given grain size in elongated glass tubes, 8 mm in diameter, is damped as a rule without attaining the end of the tube, if it has been initiated by a detonator, and expands through the whole length of the tube, if initiated by a piece of detonating fuse.

It is of some interest to mention further that when the detonation passes from a wide tube into a narrow one (both tubes filled up with the same substance) its expansion in the narrow portion of the charge is stable at such diameters, at which any other method of initiation would produce a damped detonation.

An analysis of the conditions favourable to the «super-initiation» effect leads to a preliminary conclusion that this phenomenon takes place in such cases, when the diameter of the charge approaches the maximum diameter at which the detonation may yet expand in the given substance under the given conditions. The cause of the considered phenomenon is not quite clear. It may be supposed that under certain conditions the initiator stimulates such thickening of the substance as to render it incapable of detonating. This assumption cannot, however, explain the reason why in case of «excessive» initiation the damping of detonation does not begin at once at the point of contact with the initiator, but develops gradually, and the complete ceasing of detonation occurs at a distance of several

diameters at the least. As already mentioned above, the «super-initiation» is observed solely in the case of elongated charges; the detonation velocity in the vicinity of a more powerful initiator is by far higher than in the case of a less powerful one, securing a stable expansion of detonation.

According to the present ideas adopted in the text-books on the theory of explosives, the mechanism of initiation is reduced to a shock delivered to the initiated substance by the gases of the initiator. It is beyond all doubt that a more powerful initiator (for instance, an additional charge of tetryl instead of the detonator) delivers a stronger shock to the initiated substance. The fact that under certain conditions, no matter how rare, this shock cannot secure a stable detonation, though a weaker «shock» from a less powerful initiator can, brings us to a natural conclusion that the current interpretation of the initiation mechanism is inadequate.

Further investigation is necessary on the problem of the mechanism of initiation.

Institute of Chemical Physics.
Academy of Sciences of the USSR.

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BIOCHEMISTRY

CONDITIONS NECESSARY FOR THE ACTIVITY OF CHLOROPLASTS OUTSIDE THE CELL

By E. A. BOICHENKO

(Communicated by L. A. Orbeli, Member of the Academy, 10.XI.1942)

The isolation of assimilating chloroplasts from a vegetable cell would make it possible directly to inquire into the process of photosynthesis unaffected on the part of the living protoplasm. The effect of a given factor upon the mechanism of photosynthesis has usually been studied under conditions of an indirect influence of the altered cytoplasm on the chloroplasts. The variations called forth in this way cannot be evaluated without isolating the whole assimilating complex. All attempts to reproduce photosynthesis with the help of various preparations of chlorophyll or chloroplastine proved a failure. About the middle of the thirties the attention of students was focused upon the very chloroplasts, the methods of their isolation and upon tests of their activity outside the cell.

As early as in the last century Engelmann, Haberlandt, Ewart and Kny, who were the first to isolate chloroplasts by tearing or cutting leaves in saccharose solutions, showed separate isolated chlorophyll grains to evolve oxygen during a short period of time. Later experiments by Beijerinck and Molisch with extracts from leaves, ground fresh or carefully dried, went to support these data. The evolution of oxygen, however, was no intense one, and it lasted no more than a few hours after the chlorophyll grains had been isolated from the cell. Willstätter and Stoll, as well as Inman ([1]) in his later work, have failed to observe absorption of carbon dioxide, nor was a reduction of carbon dioxide noticed in Hill's experiments where isolated chloroplasts were able to evolve oxygen in the light during several hours from iron oxalate, thereby reducing the trivalent iron to bivalent one ([2]).

From an analysis of the composition of media that had been used for the isolation of chloroplasts, it is evident that many of them match the osmotic conditions of a cell as to the content of saccharose, etc. Some experimentators have succeeded in adapting the pH of the medium to that of the intracellar one. No one of them, however, except Lubimenko, has tried to reproduce the redox conditions, to which the plastide is subjected in a living cell. Lubimenko ([3]) was the first to characterize those conditions as being anaerobic. Later on Würmser has practically shown that although in the course of assimilation there takes place evolution of oxygen, the cells are at the same time reducing indophenols whose leucoform is apt to oxidize in the air ([4]). The low rH in the vicinity of chlorophyll grains has also been pointed out by Uspensky([5]). According to this author, the cells are always producing active hydrogen from the organic matter stored by them. Near the plastides hydrogen concentration may be still higher, owing to «accumulation on the chlorophyll». On the other hand, it has been shown by van-Niel that in the case of photosynthesis by purple bacteria for the purpose of carbon

dioxide reduction may be used not only sulphur compounds (in *Thiorhodaceae*), but also molecular hydrogen or organic matter (as in *Athiorhodaceae*) (6). This author interprets photosynthesis as a redox reaction:

$$xCO_2 + 2xH_2A \rightarrow (C)_x + 2xH_2O + 2xA,$$

where H_2A denotes an hydrogen donator; (C), the product of CO_2 reduction; A, the oxidized form of the donator. Finally, Gaffron has succeeded in demonstrating that in the light green plants may use molecular hydrogen for reducing carbon dioxide (7). Photosynthesis in green plants was thus shown to be similar to that in bacteria; and it appeared probable that the former could not do without the reducing matter (H_2A) either. The fact that chloroplasts isolated from the cell fail to reduce carbon dioxide may be due to an excessively high rH which in most solutions used for the experiments amounted to 28. It appeared therefore necessary to test the activity of isolated chloroplasts at lower rH values.

The chloroplasts were obtained by grinding green leaves in a mortar, by tearing them into pieces with a micromanipulator needle, or by cutting with scissors. With the help of the first method which is now frequently used for chemical analyses of plastides, they show under the microscope an almost total disintegration. Such a material is no good for physiological tests, having already lost its activity. The use of micromanipulator needle made the work rather tedious, offering no great advantage as compared to the use of a scissor, applied in the experiments described below. Isolation of plastides was tried on algae (*Nitella*, *Vaucheria*); mosses (*Mnium*, *Polytrichum*); flowering plants (*Helodea*, *Aspidistra*, *Trifolium*, etc.). As a result, the choice fell upon *Trifolium repens*. The chloroplasts obtained from cut leaves, were collected in a 0.5 M solution of glucose in tap water, 1 cm^3 of solution per 1 g of fresh leaves. The extract was separated from intact cells by seeping it through linen or plankton silk. Further separation of chloroplasts is now usually practised by means of centrifuging. However, when centrifuged in water solution at several thousand revolutions per minute it proved impossible to prevent the chloroplasts from falling into parts; for this reason this method had to be abandoned. In order to separate intact chloroplasts from their fragments and from the solution, we practised filtration through a paper filter put into a Büchner funnel, after which the solution was sucked off by a water jet pump. From 1 cm^3 of the solution the chloroplasts were collected on 1 cm^2 of the filter. In this way there were obtained dark-green, almost black films which were dried upon the same Büchner funnel by suction, after which they could be preserved during months.

In order to check whether such films were able to reduce carbon dioxide directly, they were put into test-tubes, containing 0.1% buffer-solutions of $NaHCO_3$ with CO_2 at various pH, and exposed to bright sun. In control experiments similar films were exposed, the filter being turned to the sun with its inner surface. The reduction of carbon dioxide was estimated after the pH variations with the help of Clarke's indicators. In none of the tests have we succeeded in detecting absorption of carbon dioxide at pH as high as 5—6—7—8. This failure may possibly be accounted for by a change in the osmotic properties, the living cell containing very little water. On this account the experiments described above were repeated under addition of various amounts of saccharose ranging from 0.5 to 5 M. No absorption of carbon dioxide could yet be obtained. The work was continued with the purpose to select a convenient rH among its lowest values. In order to obtain a lower rH value, out of various reducers fructose was chosen. 0.1% solutions of Na_2CO_3, K_2CO_3, K_2HPO_4 and $Ca(OH)_2$ in saccharose were saturated with carbon dioxide until the value of pH attained 8.2. They were added 0.1% fructose, after which the chloroplasts were introduced into solution, and the test-tubes exposed to sun light. Absorption of carbon dioxide was finally detected in experiments with Ca in a 5 M saccharose solution. With Na and K salts no absorption of carbon dioxide could be detected with any of the saccharose concentrations. According to Würmser's data, the reducing action of monosaccharides depends

Author and year	Media	Method of evaluation	Evolution of O_2	Absorption of CO_2	pH	rH
Engelmann, 1881; Haberlandt, 1887; Ewart, 1896; Kny, 1897	Water and 10—25% saccharose	Movement of bacteria and indigo oxidation	1—6 hours			
Beijerinck, 1902; Molisch, 1904	Water	Fluorescence of bacteria	Several hours			
Willstätter and Stoll, 1918	$KHCO_3$ with glucose	Gasometrical analysis	—	No absorption		
Molisch, 1925; Inman, 1935—38	Water	Fluorescence of bacteria and changes in pH	From several minutes to several hours	No absorption	6.0	
Granik, 1938	0.5 M glucose	Movement of bacteria	Several minutes		6.5	
Menke, 1938	M/15 K-phosphate	Manometrical analysis	No evolution		6.8	
Hill, 1937—39; Hill and Scarisbrick, 1940	10% saccharose + $K_3Fe(C_2O_4)_3$ + + K-phosphate	Oxidation of haemoglobin	1—6 hours		7.0; 8.0	
Boichenko, 1942	0.5 — 5 M saccharose + Na_2CO_3; K_2CO_3; K_2HPO_4	Oxidation of leucodyes and changes in the pH	No evolution	No absorption	5; 6;7; 8.2	> 22.4
Do	Same + 0.1% fructose	Do	Do	Do	5; 6;7;8.2	> 9.9
»	0.5—4 M saccharose + $Ca(OH)_2$ + + fructose	»	»	»	8.2	> 5.2
»	5 M saccharose + 0.1% $Ca(OH)_2$ + + 0.1% fructose	»	Evolution up to total disintegration of carbon dioxide	Absorption of carbon dioxide within an unlimited time	8.2 → 8.5	< 5.2
»	75% (Ac)Mg(OH) + 0.1% fructose	»			7.5 → 8.5	< 5.2

on their enolization. Calcium hydroxide has been shown by Kuzin readily to give enolic forms; this is why it served as a good catalyzer in the production of sugars from formaldehyde([8]), whereas salts of alkaline metals cannot serve as such catalyzers. In the subsequent work the solutions of saccharose with calcium hydroxide were replaced by a saturated solution of basic magnesium acetate. The solution was prepared according to Kuzin; the transparent syrup obtained gave no precipitate upon addition of 0.1% of carbon dioxide, since there was scarcely any dissociation of the basic salt. Upon addition of 0.1% of fructose the limit potential was gradually established. In this way it was made possible to study the behaviour of chloroplasts at different rH in solutions of identical composition. The variations in the rH were evaluated after the reduction of the scale of the following indicators: dichlorophenylindophenol (22.4), methylenblue (14.4), indigocarmin (9.9), Janusgreen (5.2).

With rH values below 5 there was absorption of carbon dioxide with a simultaneous evolution of oxygen; the latter caused secondary oxidation of the leucodyes in the upper part of the test-tubes. Near the surface of the films, however, the rH remained throughout the exposure lower than in the controls. The exposed chloroplasts acted thereby like catalyzers of platinum black, in the presence of which the rH of sugar solutions may drop as low as the atmosphere of molecular hydrogen([9]). The fact of a total lack of reduction of carbon dioxide by the chloroplasts alone, without addition of fructose, should be taken to mean that the photosynthesis in green plants, similar to that in bacteria, requires the presence of reducing substances that would create near the exposed surface of chloroplasts a low rH. Active forms of monosaccharides may act as such reducers. In green plants the donators of hydrogen H_2A from van-Niel's photosynthesis equation are thus identical with their products of reduction of carbon dioxide $(C)_x$. Unlike photosynthesis in bacteria in which the process terminates as soon as the donators of hydrogen become oxidized, in green plants it may go on during an unlimited period of time. According to Würmser, the oxidized form of sugar A may easily pass into an active enolic form under anaerobic conditions. In the case of an increasing amount of reducing sugars near the chloroplasts, where there are concentrated most of the lipoids of the vegetable cell and very little water, the conditions are similar to those prevailing in experimental solutions. They can prevent from an increase of the rH value even in the case of oxygen evolution.

Laboratory for Biogeochemistry.
Academy of Sciences of the USSR.

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